J. Fluid Mech. (1986), *vol.* 165, *pp.* 273–296
Printed in Great Britain

Wave interaction between adjacent slender bodies

By S. R. BREIT AND P. D. SCLAVOUNOS
Massachusetts Institute of Technology, Department of Ocean Engineering, Cambridge, Massachusetts, USA

(Received 12 June 1985)

A linear approximation for surface-wave radiation by two adjacent slender bodies is derived and compared with a three-dimensional numerical method. The approximation incorporates slender-body theory for a single body and accounts for wave interaction between the bodies. It is assumed that the distance between the bodies is on the order of their lengths. The far-field disturbance due to each body is obtained by distributing wave sources and dipoles on its centreline and solving a pair of coupled integral equations for their strengths and moments respectively. The hydrodynamic added-mass and damping coefficients are then calculated from simple expressions involving the source strengths and the hydrodynamic coefficients of each body separately. Wave exciting forces are also calculated from a far-field reciprocity relation. The approximation performs well even when the separation distance is comparable to the characteristic transverse dimension of each body.

1. Introduction

Predicting the wave-induced motions of a twin-hull ship is a practical problem involving wave interaction between adjacent slender bodies. Closely related situations include two ships transferring cargo at sea or a conventional ship operating near a harbour or a channel boundary. Periodic structures such as the piers of a bridge can be modelled as an array of slender bodies. In all of these cases, first-order wave forces can be determined by neglecting viscous effects, linearizing the boundary conditions and decomposing the problem into radiation and diffraction components. Numerical solutions of the associated linear boundary-value problems by boundary-integral or finite-element methods are possible, but the computational effort can be prohibitive when multiple bodies are present. Aside from these general numerical techniques, exact accounts of wave interaction among an array of vertical, axisymmetric cylinders have been derived first in the context of acoustics by Twersky (1952), and later for surface waves by Spring & Monkmeyer (1974), and Miles (1983). Approximate treatments of wave interaction among non-axisymmetric bodies separated by sufficiently large distances have been presented by Ohkusu (1974), Greenhow (1980), Simon (1982), and Kagemoto & Yue (1985). Ohkusu (1974), Srokosz & Evans (1979), and Martin (1984) have approximated wave interaction between two-dimensional bodies that are far apart. Typically, the multiple-body problem is reduced to a set of radiation–diffraction problems for each body separately, the solutions of which are then combined analytically to approximate the wave-interaction effects.

A different class of approximate methods has evolved for treating radiation and diffraction by conventional mono-hull ships. The slenderness of the ship geometry justifies approximating the flow near the ship by a sequence of two-dimensional problems. Pioneered in naval architecture by Korvin-Kroukovsky (1955), this approach is known as 'strip theory' and has undergone a number of refinements primarily aimed at accounting for forward-speed effects. Ogilvie & Tuck (1969) presented a rational justification of strip theory and showed that the absence of longitudinal hydrodynamic interaction can be formally justified for high frequencies, corresponding to wavelengths comparable to the ship beam. The high-frequency restriction has been removed in the 'unified theory' derived by Newman (1978) and Sclavounos (1984) where the strip-theory solution near the ship is supplemented by a homogeneous component which accounts for longitudinal hydrodynamic interaction. The strength of the homogeneous component is determined by using the technique of matched asymptotic expansions.

Unified theory is applied in the present study to solve the zero-speed mono-hull problem, and is extended to include wave interaction between adjacent slender bodies with no forward speed. To simplify the derivation, we consider two identical hulls which are symmetric about their own centreplanes and rigidly connected in a catamaran configuration. Generalizations to cases where the hulls do not satisfy all of these conditions present no additional fundamental difficulties. For the sake of generality, the hulls will hereafter be referred to as bodies.

Intuitively, the interactions between two bodies can be viewed as follows. Each body radiates waves due to its own oscillatory motion as if the other body were not present. Some of the waves radiate to infinity and some interact with the adjacent body. The steady-state oscillatory solution may be viewed as the limit of a flow that started from rest and evolved through an infinite number of wave reflections. This approach calls for the treatment of an infinite series of single-body reflection/transmission problems, the summation of which leads to the steady-state flow, and has been used by Ohkusu (1974) to approximate interaction between two-dimensional bodies and among an array of vertical cylinders in three dimensions. In the two-dimensional case, Ohkusu assumes that the bodies are far enough apart to neglect non-wavelike behaviour and sums the infinite series analytically to obtain simple expressions for the hydrodynamic coefficients and exciting forces. Non-wavelike behaviour is accounted for in the three-dimensional case, but it is not possible to sum the infinite series.

An alternative view of wave interaction is typified by Simon (1982). The wave disturbance in the vicinity of each body is the sum of a radiation component resulting from its own single-body radiation, an 'incident wave' representing the influence of the adjacent body, and a diffraction component corresponding to that incident wave. The amplitude of the incident wave is *a priori* unknown and is determined by enforcing a set of compatibility conditions on the wave disturbance in the vicinity of each body. Simon approximates the cumulative incident wave by a plane wave; thus non-wavelike effects are neglected.

The three-dimensional approximations of Ohkusu and Simon are suitable only for multiple compact bodies such as vertical cylinders. For such bodies it is appropriate and convenient to assume that the wave disturbance incident on one body due to surrounding ones is generated by point radiators or scatterers. In the present problem, however, waves impinging on one slender body are generated continuously from all sections of the adjacent body, and vice versa. A simplified view of the flow would suggest that interaction occurs between body sections which lie in the same transverse plane. The three-dimensional problem is thereby reduced to a sequence of two-dimensional problems distributed along the lengths of the bodies. This strip-like approach allows wave energy to flow only in the transverse direction, and consequently exaggerates interaction effects near the resonant frequencies of oscillation where wave energy is trapped between the two bodies.

Clearly, three-dimensional effects play a role in the dissipation of wave energy reflecting between two slender bodies. The present study accounts for longitudinal interactions among the sections of each body as in unified theory, and interactions of all sections of one body with all sections of the other. The far-field wave disturbance due to each body is modelled by distributing three-dimensional wave sources and horizontal dipoles on its axis, keeping both wavelike and non-wavelike terms in their definitions. At each section of one body, the far-field disturbance of the other body is approximated by a plane wave propagating perpendicularly to its axis. Unlike strip theory, however, the longitudinal distribution of wave amplitude depends on the initially unknown source-strength and dipole-moment distributions. These distributions are found by solving a pair of coupled integral equations, obtained by asymptotic matching techniques, with the forcing terms being the unified-theory solutions of the single-body radiation problems.

A formal statement of the linearized wave–body-interaction problem is given in §2. In §3 we introduce the principal slender-body approximations and summarize the derivation of unified theory for the radiation and beam-wave-diffraction problems. The solution of the twin-body wave-interaction problem is presented in §4. It is shown that by allowing the body length to increase, while the frequency is kept constant, the two-dimensional approximation derived by Ohkusu (1974) is recovered. Section 5 outlines the numerical solution of the integral equation and the subsequent evaluation of the heave and pitch added-mass and damping coefficients and exciting forces. The latter are obtained by deriving a far-field reciprocity relation between the solutions of the radiation and diffraction problems. Computations of the hydrodynamic coefficients and exciting forces are compared in §6 with results from a very accurate three-dimensional panel method. Extensions of the technique to more general cases are discussed in §7.

2. Problem formulation

We consider a floating body which is free to oscillate about a fixed mean position. The translational and rotational motions are defined with respect to a Cartesian coordinate system where the x- and y-axes coincide with the free surface and the z-axis is oriented upwards. The fluid domain has infinite depth and horizontal extent. We assume that the fluid is inviscid and incompressible, and the fluid motion irrotational; and so we may pose a boundary-value problem in terms of a scalar velocity potential.

The boundary conditions can be applied at the mean positions of the free surface and the body boundary if the amplitudes of the ambient waves and body motions are small relative to a characteristic dimension of the body and the wave amplitude is small relative to the wavelength. The boundary-value problem for the velocity potential can then be solved in the frequency domain by factoring out the oscillatory time-dependence $e^{i\omega t}$, where ω is the circular frequency and t is time, and solving for the complex potential $\phi(x, y, z)$ which is time-independent. Henceforth, the oscillatory time dependence will be implicit. Any physical quantity can be obtained by taking the real part of the product of the corresponding complex quantity and the oscillatory time dependence.

The complex velocity potential ϕ must satisfy

$$\nabla^2\phi = \left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\phi = 0 \quad \text{in } \mathscr{V}, \tag{2.1}$$

$$\phi_z - K\phi = 0 \quad \text{on } z = 0, \tag{2.2}$$

$$\nabla\phi \rightarrow 0 \quad \text{as } z \rightarrow -\infty, \tag{2.3}$$

where $\mathscr{V}$ denotes the fluid domain, $K = \omega^2/g$ is the wavenumber in water of infinite depth and g is the acceleration due to gravity.

The body boundary condition is easier to treat after making the linear decomposition

$$\phi = A(\phi_0+\phi_7)+\sum_{j=1}^{6} \phi_j \xi_j, \tag{2.4}$$

where A is the amplitude of the free-surface elevation due to the incident wave and ξ_j ($j = 1, \ldots, 6$) are the complex amplitudes of the rigid-body oscillations. The modes $j = 1, 2, 3$ correspond to translation in the x-, y- and z-directions and modes $j = 4, 5, 6$ to rotation about the same axes respectively. The normalized velocity potentials ϕ_0, ϕ_j, and ϕ_7 govern the incident, radiated and diffracted wave flows.

Regular deep-water plane-progressive waves are described by the velocity potential

$$\phi_0 = \mathrm{i}\frac{g}{\omega} \exp\left[K(z-\mathrm{i}x \cos\beta-\mathrm{i}y \sin\beta)\right], \tag{2.5}$$

where β is the angle between their direction of propagation and the positive x-axis. The diffraction velocity potential ϕ_7 accounts for interaction between the body, fixed at its mean position, and the incident waves. Thus, on the body's wetted surface S_{B}, it offsets the incident-wave normal velocity

$$\boldsymbol{n}\cdot\nabla\phi_7 = -\boldsymbol{n}\cdot\nabla\phi_0 \quad \text{on } S_{\mathrm{B}}, \tag{2.6}$$

where the unit normal vector $\boldsymbol{n}$ is taken to point out of the fluid domain.

The radiated waves are generated by the forced oscillation of the body in otherwise calm water. The radiation velocity potentials satisfy the inhomogeneous conditions

$$\boldsymbol{n}\cdot\nabla\phi_j = \mathrm{i}\omega n_j \quad \text{on } S_{\mathrm{B}} \quad (j = 1, 2, \ldots, 6), \tag{2.7}$$

where the normal components n_j are defined by

$$\boldsymbol{n} = (n_1, n_2, n_3), \quad \boldsymbol{x}\times\boldsymbol{n} = (n_4, n_5, n_6). \tag{2.8}$$

Since n_j ($j = 1, \ldots, 6$) are real, the real parts of ϕ_j satisfy a homogeneous body boundary condition. This property will be exploited in the derivation of the slender-body theory outlined in the next section.

Finally, the problem is not well-posed unless the radiation and diffraction velocity potentials represent outgoing waves as $R\rightarrow\infty$, where $R^2 = x^2+y^2$. This requirement is known as the radiation condition.

3. The unified slender-body theory

We denote the length of a body by L, its characteristic transverse dimension by B and assume the slenderness parameter $\epsilon = B/L$ to be small compared to unity. If the longitudinal axis coincides with the x-axis and $L = O(1)$, then near the body $x = O(1)$ and $y, z = O(\epsilon)$. Scaling arguments suggest that, in the near field, the three-dimensional problem can be replaced by a collection of two-dimensional problems in transverse planes. This approximation is not valid in the far field where the flow is still three-dimensional. The unified theory developed by Newman (1978) accounts for longitudinal hydrodynamic interaction by enforcing the compatibility of the two-dimensional near-field solutions and the three-dimensional far-field solution. This section summarizes the unified-theory solutions of the radiation and the beam-wave diffraction problems for a single body in a form that will be convenient for the formulation and solution of the twin-body problem presented in §4.

3.1. *Inner solution*

A general inner solution for both the radiation and diffraction problems is constructed by superimposing a particular and a homogeneous component. The particular solution $\psi(y, z; x)$ satisfies

$$\nabla^2_{2D}\psi = \left(\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\psi = 0 \quad \text{in } \mathscr{D}, \tag{3.1}$$

$$\psi_z - K\psi = 0 \quad \text{on } z = 0, \tag{3.2}$$

$$\nabla_{2D}\psi \to 0 \quad \text{as } z \to -\infty, \tag{3.3}$$

and a condition of outgoing waves for large $K|y|$.

In the two-dimensional radiation problem, only modes $j = 2, 3, 4$ are meaningful. The corresponding velocity potentials satisfy the inhomogeneous body boundary conditions

$$\boldsymbol{n}\cdot\nabla_{2D}\psi_j = \mathrm{i}\omega n_j \quad \text{on } C(x) \quad (j = 2, 3, 4), \tag{3.4}$$

where C is the body cross-section in a constant-x plane. Solutions for the rotational pitch and yaw modes ($j = 5, 6$ respectively) are defined in terms of the two-dimensional heave and sway solutions by the relations

$$\psi_5(y, z; x) = -x\psi_3(y, z; x), \tag{3.5a}$$

$$\psi_6(y, z; x) = x\psi_2(y, z; x), \tag{3.5b}$$

where terms quadratic in the body slenderness have been neglected.

The velocity potential of a unit-amplitude wave propagating in the positive-y direction is given by

$$\psi_0 = \mathrm{i}\frac{g}{\omega}\mathrm{e}^{K(z-\mathrm{i}y)}. \tag{3.6}$$

The sum of the incident-wave and diffraction potentials must therefore satisfy

$$\boldsymbol{n}\cdot\nabla_{2D}(\psi_0+\psi_7) = 0 \quad \text{on } C(x). \tag{3.7}$$

The particular solutions may be decomposed into parts which are symmetric and antisymmetric with respect to the longitudinal centreplane $y = 0$. Henceforth, we will use s and a subscripts to denote the symmetric and antisymmetric parts of these solutions respectively.

The homogeneous components of the near-field solution account for hydrodynamic interaction between adjacent transverse sections. Newman (1978) has shown that, for the antisymmetric modes of motion, the strength of the interaction is of higher order and, to leading order, the inner solution is given by the particular component. In the radiation problem, homogeneous solutions for the symmetric modes are the real parts of the velocity potentials ψ_j ($j = 3, 5$). This can be easily verified from the fact that the boundary conditions (3.4) are purely imaginary. In the diffraction problem, a homogeneous solution is simply the sum of the incident-wave and diffraction potentials. We denote the general solutions of the radiation and diffraction problems by ϕ_j and ϕ_7 respectively, and write them in the form

$$\phi_j(x, y, z) = \psi_j(y, z; x) + C_j(x)\,(\psi_{j,\,\mathrm{s}} + \psi^*_{j,\,\mathrm{s}}), \tag{3.8a}$$

$$\phi_7(x, y, z) = \psi_7(y, z; x) + C_7(x)\left(\mathrm{i}\frac{g}{\omega}\,\mathrm{e}^{Kz}\cos Ky + \psi_{7,\,\mathrm{s}}\right), \tag{3.8b}$$

where ψ_j^* denotes the complex conjugate of ψ_j; C_j and C_7 are unknown interaction coefficients which depend parametrically on the x-coordinate. They will be determined from the asymptotic matching with the outer solution.

In order to match the inner and outer solutions, far-field asymptotic expansions of the inner solutions are required. We denote by $G_{2D}(y, z)$ the velocity potential due to an oscillatory wave source located at the origin of the coordinate system. This potential is also known as the two-dimensional Green function and is derived in Wehausen & Laitone (1960). It satisfies (3.1)–(3.3) and a condition of outgoing waves for large $K|y|$. The potential of an oscillatory horizontal dipole, which we denote by $H_{2D}(y, z)$, is obtained from the relation

$$H_{2D}(y,z) = \frac{1}{K}\frac{\partial}{\partial y} G_{2D}(y,z). \tag{3.9}$$

Stated in terms of the source and dipole potentials, the outer expansions of the particular solutions are

$$\psi_j(y,z;x) \simeq \sigma_j(x)\, G_{2D}(y,z) + \mu_j(x)\, H_{2D}(y,z) \quad (j = 1,2,\ldots,6), \tag{3.10a}$$

$$\psi_7(y,z;x) \simeq \sigma_7(x)\, G_{2D}(y,z) + \mu_7(x)\, H_{2D}(y,z), \tag{3.10b}$$

where σ_j, σ_7 are complex source strengths and μ_j, μ_7 are complex dipole moments. Henceforth, we shall assume these quantities are known at each transverse section of the ship. This implies that the particular solutions are known at a continuum of longitudinal locations. In practice, of course, the two-dimensional problem is solved at a finite number of sections. We have assumed that the bodies are symmetrical with respect to the $y = 0$ plane; therefore either the source strength or dipole moment is zero for a particular mode of motion. We will neglect this fact and continue as if both the source and dipole are present.

Substituting (3.10a, b) into (3.8a, b) respectively gives the complete outer expansions of the inner velocity potentials

$$\begin{aligned}\phi_j \simeq {} & [\sigma_j + C_j(\sigma_j + \sigma_j^*)]\, G_{2D} - \mathrm{i} C_j\, \sigma_j\, \mathrm{e}^{Kz} \cos Ky \\ & + \mu_j\, H_{2D},\end{aligned} \tag{3.11a}$$

$$\begin{aligned}\phi_7 \simeq {} & (1 + C_7)\, \sigma_7\, G_{2D} + \mathrm{i} C \frac{g}{7\omega}\, \mathrm{e}^{Kz} \cos Ky \\ & + \mu_7\, H_{2D},\end{aligned} \tag{3.11b}$$

where we have used the identity

$$G_{2D}^* = G_{2D} - \mathrm{i}\, \mathrm{e}^{Kz} \cos Ky. \tag{3.12}$$

The outer expansions have been purposely displayed on two lines. The terms on the first line correspond to the symmetric part of the disturbance while the term on the second line corresponds to the antisymmetric part. We will continue this practice throughout the remainder of this paper.

3.2. *Outer solution*

The symmetric and antisymmetric parts of the potential are represented in the far field by distributions of three-dimensional oscillatory sources and horizontal dipoles respectively, on the body's centreline. We denote the three-dimensional oscillatory source potential by $G(x - \xi, y, z)$, where $(\xi, 0, 0)$ is the source point and (x, y, z) is the field point. It satisfies (2.1)–(2.3) and a condition of outgoing waves at infinity. The potential of an oscillatory dipole oriented in the positive-y direction, which we denote by $H(x-\xi, y, z)$, is obtained from a relation which is the three-dimensional analogue of (3.9). Using these definitions, the radiation and diffraction solutions are approximated in the far field in the form

$$\phi_j(x,y,z) = \int_L \mathrm{d}\xi\, q_j(\xi)\, G(x-\xi, y, z) + \int_L \mathrm{d}\xi\, d_j(\xi)\, H(x-\xi, y, z) \quad (j = 1, 2, \ldots, 7), \tag{3.13}$$

where q_j and d_j are, respectively, complex source strength and dipole moment distributions which are initially unknown, and L indicates that the range of integration is over the length of the body.

In order to facilitate the asymptotic matching with the inner solution, an inner expansion of the outer solution must be obtained in terms of $G_{2\mathrm{D}}$. Such an expression is given by

$$\phi_j(x,y,z) \simeq q_j(x)\, G_{2\mathrm{D}}(y,z) - \mathrm{L}_u(q_j; x) + d_j(x)\, H_{2\mathrm{D}}(y,z) + O(Kr) \quad (j = 1, 2, \ldots, 7), \tag{3.14}$$

where $r = (y^2+z^2)^{\frac{1}{2}}$ and

$$\mathrm{L}_u(q;x) = \tfrac{1}{2}(\mathrm{i}+\gamma/\pi)\, q(x) + \frac{1}{4\pi}\int_L \mathrm{d}\xi\, q'(\xi)\, \mathrm{sgn}\,(x-\xi) \log(2K|x-\xi|) - \tfrac{1}{8}K \int_L \mathrm{d}\xi\, q(\xi)\, [Y_0(K|x-\xi|) + 2\mathrm{i}J_0(K|x-\xi) + \boldsymbol{H}_0(K|x-\xi|)] \tag{3.15}$$

is a linear integral operator on $q(x)$.

3.3. *Matching*

The outer expansions of the inner solutions can be matched with the inner expansions of the outer solutions in some intermediate domain. The leading-order terms in (3.11) and (3.14) for small r are the dipole potentials which behave like $1/r$. Equating the factors of the dipole potentials gives

$$d_j = \mu_j. \tag{3.16a}$$

$$d_7 = \mu_7. \tag{3.16b}$$

As we have already mentioned, the remaining antisymmetric terms in the inner and outer expansions are of higher order. Therefore the radiation solutions for antisymmetric modes and the antisymmetric part of the diffraction solution are strictly two-dimensional in the inner region.

Next we match the symmetric terms in (3.11) and (3.14). The leading-order symmetric terms are the source potentials which behave like $\log r$. Equating the factors of the source potentials gives the relations

$$q_j = \sigma_j + C_j(\sigma_j + \sigma_j^*), \tag{3.17a}$$

$$q_7 = \sigma_7 + C_7\, \sigma_7. \tag{3.17b}$$

Equating the remaining terms in (3.11) and (3.14) which are $O(1)$, we obtain

$$\mathrm{L}_u(q_j; x) = +\mathrm{i}C_j\, \sigma_j, \tag{3.18a}$$

$$\mathrm{L}_u(q_7; x) = -\mathrm{i}\frac{g}{\omega} C_7. \tag{3.18b}$$

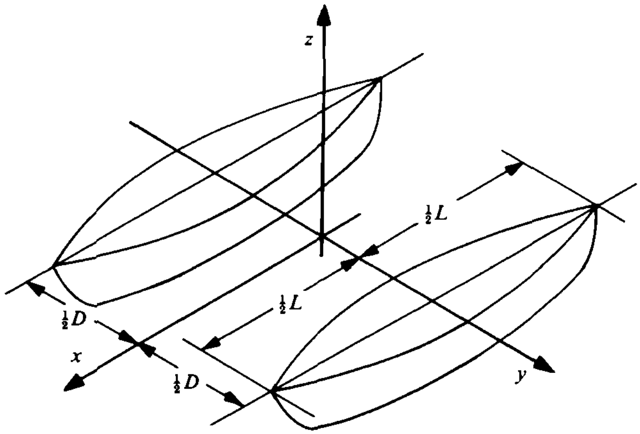

FIGURE 1. Geometric configuration of two adjacent slender bodies.

In practice, (3.18a, b) are used to eliminate C_j and C_7 from (3.17a, b) respectively. This leads to linear integral equations for q_j and q_7. Further details on the theory and errors involved in the inner and outer expansions are given in Newman (1978).

4. Far-field approximation for twin slender bodies

Next we consider two identical slender bodies arranged in a catamaran configuration as illustrated in figure 1. We require the bodies to be far enough apart to permit neglect of non-wavelike hydrodynamic interaction. Quantities associated with the body on the positive or negative side of the $y = 0$ plane are denoted by + or − superscripts respectively. The total velocity potential associated with S_B^+ (S_B^-) is $\phi^+ + \phi_I^+$ ($\phi^- + \phi_I^-$), where ϕ^+ (ϕ^-) is the single-body radiation disturbance and ϕ_I^+ (ϕ_I^-) accounts for interaction with S_B^- (S_B^+). The radiation disturbances are known from solving the radiation problem for each body separately, but the interaction potentials are initially unknown.

4.1. *Outer solution*

At distances which are large compared to its beam, the interaction disturbance caused by S_B^+ is represented in the form

$$\phi_I^+ = \int_L \mathrm{d}\xi\, q_I^+(\xi)\, G(x-\xi, y-\tfrac{1}{2}D, z) + \int_L \mathrm{d}\xi\, d_I^+(\xi)\, H(x-\xi, y-\tfrac{1}{2}D, z), \tag{4.1}$$

where the 'interaction' source strength q_I^+ and dipole moment d_I^+ are unknown. Similarly, the total far-field disturbance caused by S_B^- is

$$\phi^- + \phi_I^- = \int_L \mathrm{d}\xi\, [q^-(\xi) + q_I^-(\xi)]\, G(x-\xi, y+\tfrac{1}{2}D, z) - \int_L \mathrm{d}\xi\, [d^-(\xi) + d_I^-(\xi)]\, H(x-\xi, y+\tfrac{1}{2}D, z), \tag{4.2}$$

where q_I^- and d_I^- are also unknown, but q^- and d^- are known from solving the unified-theory integral equation for each body. The bodies are equidistant from the $y = 0$ plane; therefore the total potential must be either symmetric or antisymmetric about that plane, depending on the mode of motion. This means that the source strengths and dipole moments must satisfy the conditions

$$q^- = \pm q^+, \quad d^- = \mp d^+, \tag{4.3a}$$

$$q_{\mathrm{I}}^- = \pm q_{\mathrm{I}}^-, \quad d_{\mathrm{I}}^- = \mp d_{\mathrm{I}}^-, \tag{4.3b}$$

where the upper and lower signs apply to the symmetric and antisymmetric cases respectively. By virtue of these equations, we hereafter omit the + and − superscripts from the source strengths and dipole moments. The derivation which follows applies to the symmetric case, but the antisymmetric solution can be obtained simply by reversing the signs of the appropriate terms in the symmetric solution.

The inner expansion of (4.1) near S_{B}^+ is given by

$$\begin{aligned}\phi_{\mathrm{I}}^+(x,y,z) \simeq{}& q_{\mathrm{I}}(x)\,G_{2\mathrm{D}}(y-\tfrac{1}{2}D,0)-\mathrm{L}_u(q_{\mathrm{I}};x)\\ &+d_{\mathrm{I}}(x)\,H_{2\mathrm{D}}(y-\tfrac{1}{2}D,0)+O(Kr^+),\end{aligned} \tag{4.4}$$

where $r^+ = [(y-\frac{1}{2}D)^2+z^2]^{\frac{1}{2}}$ and $\mathrm{L}_u(q_{\mathrm{I}})$ is defined in (3.15). The above expression is obtained by substituting q_{I}, d_{I} for q_j, d_j in (3.14) and shifting the two-dimensional source and dipole origin to $(y,z) = (\frac{1}{2}D, 0)$.

4.2. *Inner solution*

The interaction wave disturbance in the vicinity of one body due to the presence of the other is, in general, a function of the x-coordinate. Its variation in the longitudinal direction depends both on the frequency of oscillation and on the separation between the bodies. In order to justify a two-dimensional inner solution, as in unified theory, the interaction disturbance must not vary more rapidly in the longitudinal direction than the geometry. This is a reasonable assumption for wavelengths comparable to the body length, as long as the separation distance is large enough to justify the omission of non-wavelike interaction. For wavelengths on the order of the body beam, the waves radiated by each body are primarily focused in the transverse direction near that body and become circular in the far field. Therefore a two-dimensional inner approximation is still reasonable if the separation distance is on the order of a few beams, but not if it is comparable to the body length. Fortunately, interactions are weak at high frequencies and large separation distances.

Thus, we assume that $(\phi^- + \phi_{\mathrm{I}}^-)$, which is due to the body S_{B}^-, can be approximated in the vicinity of S_{B}^+ by a plane wave propagating perpendicularly to its axis with an amplitude which varies slowly along its length. The potential of such a wave is given by

$$\phi^-(x,y,z)+\phi_{\mathrm{I}}^-(x,y,z) \simeq \mathrm{i}\frac{g}{\omega}\alpha^+(x)\,\mathrm{e}^{K[z-\mathrm{i}(y-\frac{1}{2}D)]}, \tag{4.5}$$

where α^+ is the complex amplitude which depends parametrically on the x-coordinate. It is determined by Taylor-expanding (4.2) and (4.5) about $(y,z) = (\frac{1}{2}D, 0)$ and equating the leading-order terms:

$$\begin{aligned}\alpha^+(x) = -\mathrm{i}\frac{\omega}{g}\Big\{&\int_L \mathrm{d}\xi\,[q(\xi)+q_{\mathrm{I}}(\xi)]\,G(x-\xi,D,0)\\ &-\int_L \mathrm{d}\xi\,[d(\xi)+d_{\mathrm{I}}(\xi)]\,H(x-\xi,D,0)\Big\}+O(Kr^+).\end{aligned} \tag{4.6}$$

The numerical results to be presented in §6 provide ample justification for this approximation.

The inner solution for S_{B}^{+} follows from the corresponding solution (3.8*b*) of the beam-wave diffraction problem in the form

$$\phi_{\mathrm{I}}^{+}(x,y,z) = \alpha^{+}(x)\,\psi_7^{+}(y,z) + C_{\mathrm{I}}^{+}(x)\left[\mathrm{i}\frac{g}{\omega}\mathrm{e}^{Kz}\cos K\,(y-\tfrac{1}{2}D) + \psi_{7,s}^{+}\right], \tag{4.7}$$

where the unknown coefficient C_{I}^{+} accounts for longitudinal hydrodynamic interaction. The outer expansion of the inner solution is obtained in the same manner as (3.11*b*) and is given by

$$\begin{aligned}\phi_{\mathrm{I}}^{+} \simeq{}& (\alpha^{+}+C_{\mathrm{I}}^{+})\,\sigma_7^{+}\,G_{2\mathrm{D}}(y-\tfrac{1}{2}D,0) + \mathrm{i}C_{\mathrm{I}}^{+}\frac{g}{\omega}\,\mathrm{e}^{Kz}\cos K\,(y-\tfrac{1}{2}D)\\ &+\alpha^{+}\mu_7^{+}\,H_{2\mathrm{D}}(y-\tfrac{1}{2}D,0) + O(Kr^{+}).\end{aligned} \tag{4.8}$$

4.3. *Matching*

The inner expansion (4.4) can be matched to the outer expansion (4.8) to obtain a set of equations for the unknowns q_{I}, d_{I} and C_{I}^{+}. Equating the leading-order antisymmetric and symmetric terms, we have

$$d_{\mathrm{I}} = \alpha^{+}\mu_7^{+}, \tag{4.9a}$$

$$q_{\mathrm{I}} = \alpha^{+}\sigma_7^{+} + C_{\mathrm{I}}^{+}\,\sigma_7^{+}. \tag{4.9b}$$

Equating the order-one terms gives

$$-\mathrm{L}_u(q_{\mathrm{I}}) = \mathrm{i}C_{\mathrm{I}}^{+}\frac{g}{\omega}. \tag{4.9c}$$

After using (4.9*c*) to eliminate C_{I}^{+} from (4.9*b*), we have

$$d_{\mathrm{I}} = \alpha^{+}\mu_7^{+}, \tag{4.10a}$$

$$q_{\mathrm{I}} = \alpha^{+}\sigma_7^{+} + \mathrm{i}\frac{\omega}{g}\sigma_7^{+}\,\mathrm{L}_u(q_{\mathrm{I}}). \tag{4.10b}$$

Next we define a dimensionless inner source strength and dipole moment by

$$s_7 = -\mathrm{i}\frac{\omega}{g}\sigma_7, \tag{4.11a}$$

$$m_7 = -\mathrm{i}\frac{\omega}{g}\mu_7, \tag{4.11b}$$

and the linear operators

$$\mathrm{L}_G^{-}(q;x) = \int_L \mathrm{d}\xi\, q(\xi)\, G(x-\xi,D,0), \tag{4.12a}$$

$$\mathrm{L}_H^{-}(d;x) = \int_L \mathrm{d}\xi\, d(\xi)\, H(x-\xi,D,0). \tag{4.12b}$$

By substituting (4.6) in (4.10*a*,*b*) and using the above definitions, we obtain the coupled integral equations

$$(1+s_7\,\mathrm{L}_u - s_7\,\mathrm{L}_G^{-})\,q_{\mathrm{I}} \quad + \quad (s_7\,\mathrm{L}_H^{-})\,d_{\mathrm{I}} = s_7[\mathrm{L}_G^{-}(q) - \mathrm{L}_H^{-}(d)], \tag{4.13a}$$

$$(-m_7\,\mathrm{L}_G^{-})\,q_{\mathrm{I}} + (1+m_7\,\mathrm{L}_H^{-})\,d_{\mathrm{I}} = m_7[\mathrm{L}_G^{-}(q) - \mathrm{L}_H^{-}(d)], \tag{4.13b}$$

with q and d having been determined from the solution of the single-body problem. One unknown can be eliminated by multiplying (4.13*a*) and (4.13*b*) by m_7 and s_7

respectively and then subtracting one from the other. This leads to the relation

$$d_{\mathrm{I}} = \left[\frac{q_{\mathrm{I}}}{s_7} + \mathrm{L}_u(q_{\mathrm{I}})\right] m_7, \tag{4.14}$$

which can be substituted into (4.13*a*) to obtain a single integral equation for q_{I}:

$$[1 - s_7\,\mathrm{L}_G^- + m_7\,\mathrm{L}_H^- + s_7\,\mathrm{L}_u(1 + m_7\,\mathrm{L}_H^-)]\, q_{\mathrm{I}}(x) = s_7[\mathrm{L}_G^-(q) - \mathrm{L}_H^-(d)]. \tag{4.15}$$

Equations (4.14) and (4.15) are the principal result of our analysis.

A special case of (4.13*a, b*) deserves some attention. Assume that the body geometries have a uniform sectional shape in the longitudinal direction and that their lengths tend to infinity while the separation between the bodies and the frequency of oscillation are kept constant. In the limit, the problem reduces to the purely two-dimensional one of two interacting cylinders considered by Ohkusu (1974). This special case effectively corresponds to the high-frequency limit of (4.13), since the wavelength/body-length ratio tends to zero. The operator L_u defined by (3.15), which accounts for the longitudinal hydrodynamic interaction in unified theory, tends to zero at high frequencies. Moreover, the operators defined by (4.12) reduce to the simple two-dimensional form

$$\mathrm{L}_G^-(q) = qG_{2\mathrm{D}}(D,0), \tag{4.16a}$$

$$\mathrm{L}_H^-(d) = dH_{2\mathrm{D}}(D,0), \tag{4.16b}$$

where the two-dimensional wave source and dipole have been defined in §3. Upon substituting into the coupled integral equations (4.13), they reduce to the algebraic equations

$$(1 - s_7\,\mathrm{L}_G^-)\,q_{\mathrm{I}} \;+\; (s_7\,\mathrm{L}_H^-)\,d_{\mathrm{I}} = s_7[\mathrm{L}_G^-(q) - \mathrm{L}_H^-(d)], \tag{4.17a}$$

$$(-m_7\,\mathrm{L}_G^-)\,q_{\mathrm{I}} + (1 + m_7\,\mathrm{L}_H^-)\,d_{\mathrm{I}} = m_7[\mathrm{L}_G^-(q) - \mathrm{L}_H^-(d)], \tag{4.17b}$$

with the interaction source strength q_{I} and dipole moment d_{I} as unknowns. Ohkusu has derived equations similar to (4.17) by analytically summing the infinite series obtained by posing successive reflection/transmission problems. The main difference between Ohkusu's result and the present one is that Ohkusu uses far-field approximations of the wave singularities $G_{2\mathrm{D}}$ and $H_{2\mathrm{D}}$, whereas their exact forms are kept here. This introduces non-wavelike effects into the two-dimensional approximation. The numerical results presented in §6 indicate that these effects are important at low frequencies.

5. Hydrodynamic forces

In keeping with the decomposition of the velocity potential, it is standard practice in the field of ship motions to decompose the hydrodynamic pressure force into added-mass and damping coefficients, associated with forced oscillation in various modes, and a wave-induced exciting force. The added-mass and damping coefficients a_{ij} and b_{ij} and the exciting force X_i are defined by

$$\omega^2 a_{ij} - \mathrm{i}\omega b_{ij} = -\mathrm{i}\omega\rho \iint_{S_{\mathrm{B}}} n_i\,\phi_j\,\mathrm{d}S \quad (i,j = 1,2,\ldots,6), \tag{5.1}$$

$$X_i = -\mathrm{i}\omega\rho \iint_{S_{\mathrm{B}}} n_i(\phi_0 + \phi_7)\,\mathrm{d}S \quad (i = 1,2,\ldots,6), \tag{5.2}$$

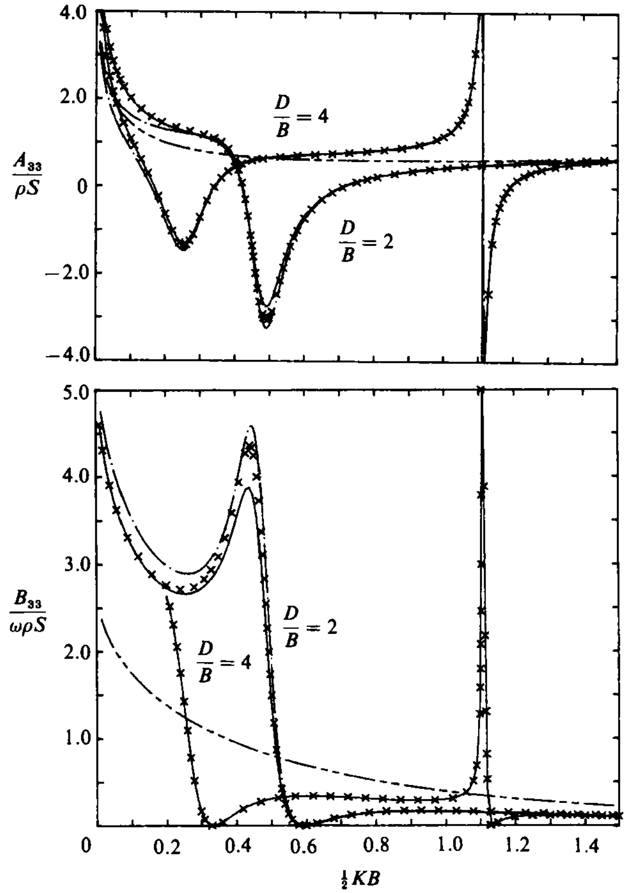

FIGURE 2. Heave added-mass and damping coefficients of twin semi-submerged circular cylinders versus wavenumber, for two separation distances D/B, where B is the cylinder diameter and D is the distance between their centres. Two versions of the two-dimensional limit of the far-field approximation are shown: with the exact form of the wave-source potential (——) and with the asymptotic form (—·—). Also shown for comparison are values from a numerical solution (Sclavounos 1985*b*) for one (– – – –) and twin (×) cylinders. The coefficients are non-dimensionalized by the fluid density ρ, the circular frequency ω, and the cross-sectional area $S = \frac{1}{8}\pi B^2$ or $S = \frac{1}{4}\pi B^2$ *of* one or two semi-submerged cylinders respectively.

where n_i is defined by (2.8) and ϕ_j, ϕ_7 are defined in §2. The above expressions come from substituting ϕ_j, ϕ_7 in the Bernoulli equation and integrating over the submerged surface to get the ith component of the pressure force on the body.

For a particular transverse section, two-dimensional coefficients A_{ij}, B_{ij} and exciting force χ_i are defined analogously by

$$\omega^2 A_{ij}(x) - \mathrm{i}\omega B_{ij}(x) = -\mathrm{i}\omega\rho \int_{C(x)} n_i \psi_j \,\mathrm{d}l \quad (i,j = 2,3,4), \tag{5.3}$$

$$\chi_i(x) = -\mathrm{i}\omega\rho \int_{C(x)} n_i(\psi_0 + \psi_7)\,\mathrm{d}l \quad (i = 2,3,4), \tag{5.4}$$

where ψ_j, ψ_7 are the particular solutions defined in §3.

Our objective is to express the added-mass, damping and exciting force for a twin-hull ship in terms of the two-dimensional properties of one hull and the solutions

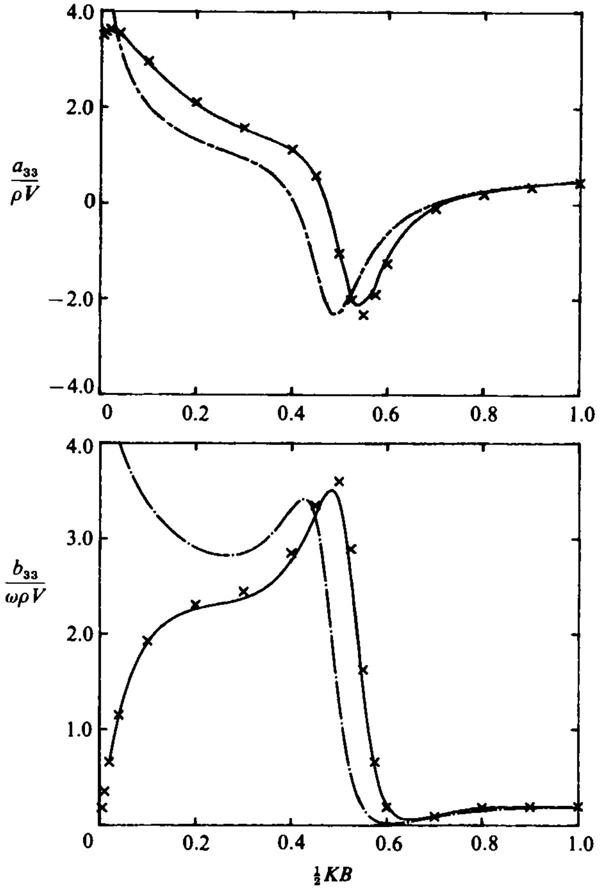

FIGURE 3. Heave added-mass and damping coefficients of twin semi-submerged spheroids with diameter-to-length ratio $\frac{1}{8}$ and separation-to-length ratio $\frac{1}{4}$; according to far-field approximation (——), strip theory (—·—) and three-dimensional numerical method (×). Coefficients are non-dimensionalized by displaced volume V of both spheroids and wavenumber K is non-dimensionalized by radius of one spheroid.

of the unified-theory integral equation and the far-field approximation for twin slender bodies. We present these relations only for the heave and pitch modes, but similar ones can be easily derived for other modes of motion. In the equations which follow, S and T superscripts denote coefficients for single and twin slender bodies respectively.

The heave and pitch coefficients of a single slender body are put in the desired form,

$$\omega^2 a_{33}^{\mathrm{S}} - \mathrm{i}\omega b_{33}^{\mathrm{S}} = \int_L \mathrm{d}x\,[\omega^2 A_{33}^{\mathrm{S}}(x) - \mathrm{i}\omega B_{33}^{\mathrm{S}}(x)] - \mathrm{i}\omega \int_L \mathrm{d}x\, C_3^{\mathrm{S}}(x)\, B_{33}^{\mathrm{S}}(x), \tag{5.5a}$$

$$\omega^2 a_{55}^{\mathrm{S}} - \mathrm{i}\omega b_{55}^{\mathrm{S}} = \int_L x^2\,\mathrm{d}x\,[\omega^2 A_{33}^{\mathrm{S}}(x) - \mathrm{i}\omega B_{33}^{\mathrm{S}}(x)] - \mathrm{i}\omega \int_L \mathrm{d}x\, C_5^{\mathrm{S}}(x)\, B_{33}^{\mathrm{S}}(x), \tag{5.5b}$$

by substituting the complete inner solution (3.8a) into (5.1) and using the definitions (5.3) and (3.5a). The first integral in each of the above equations corresponds to the strip-theory contribution and the second term is a correction which accounts for longitudinal interaction.

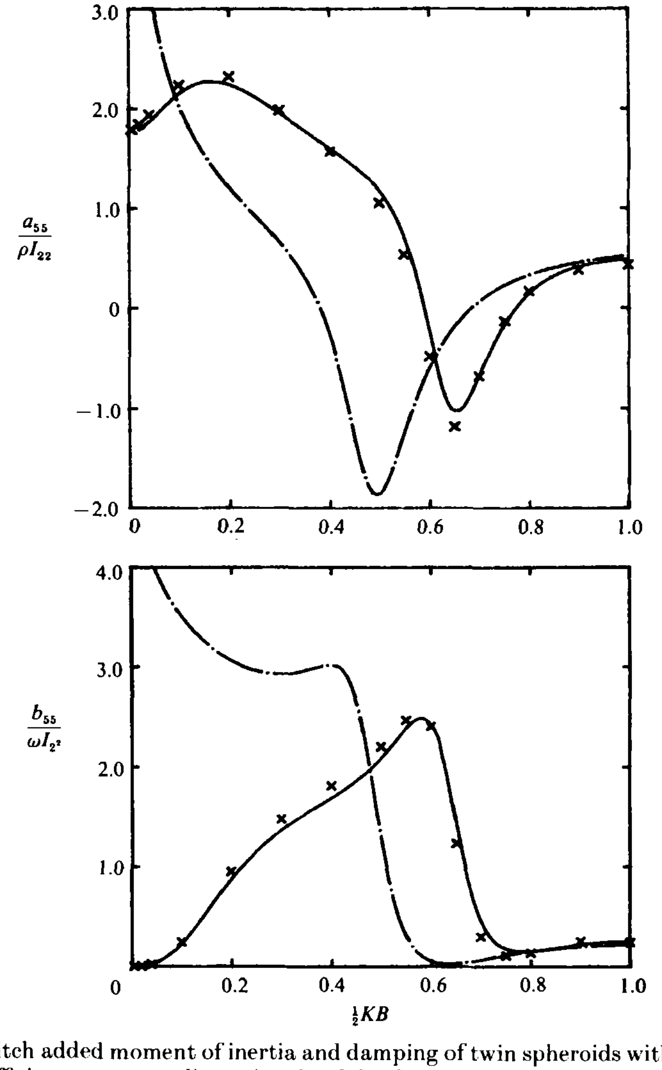

FIGURE 4. Pitch added moment of inertia and damping of twin spheroids with separation-to-length ratio $\frac{1}{4}$. Coefficients are non-dimensionalized by longitudinal moment of inertia I_{22} of volume displaced by twin spheroids.

In the case of twin slender bodies, it is only necessary to integrate over one body because the integrand of (5.1) is symmetrical about the $y = 0$ plane regardless of the symmetry or antisymmetry of the flow. After substituting the inner representation (4.7) of the interaction potential in (5.1) and using the definition (5.4), the heave and pitch coefficients can be put in the form

$$\omega^2 a_{jj}^{\mathrm{T}} - \mathrm{i}\omega b_{jj}^{\mathrm{T}} = 2\left[\omega^2 a_{jj}^{\mathrm{S}} - \mathrm{i}\omega b_{jj}^{\mathrm{S}} + \int_L \mathrm{d}x \, \frac{q_{\mathrm{I},j}(x)}{\sigma_7^{\mathrm{S}}(x)} \chi_3^{\mathrm{S}}(x)\right] \quad (j = 3, 5), \tag{5.6}$$

where $q_{\mathrm{I},j}$ is known from solving the far-field approximation (4.15). The first two terms contain the contribution from forced oscillation of each body without the presence of the other and the integral accounts for interaction between the bodies.

The exciting forces can be obtained either by solving the diffraction problem directly or by using a far-field reciprocity relation to express the exciting force in mode

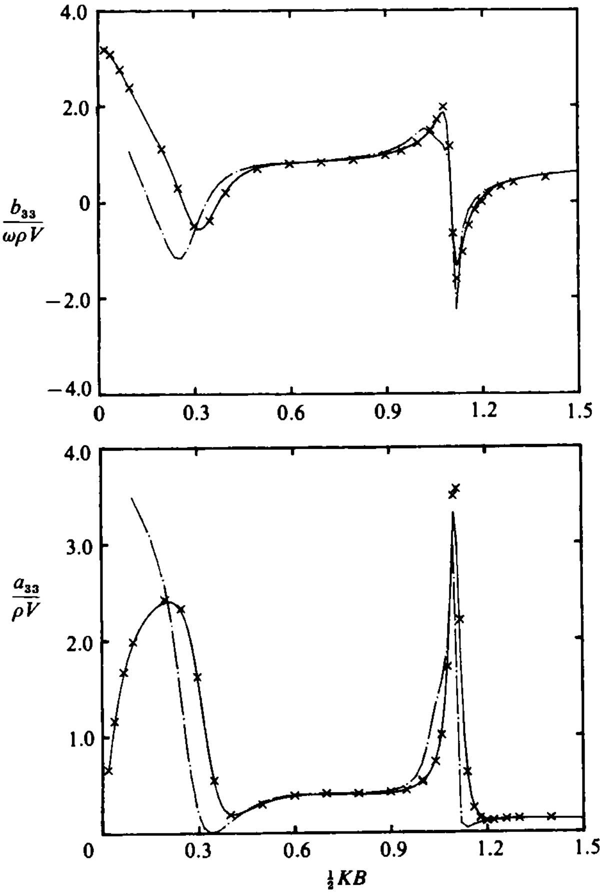

FIGURE 5. Heave added-mass and damping coefficients of twin spheroids with separation-to-length ratio $\frac{1}{2}$.

j in terms of the jth mode radiation potential. We have chosen the latter approach because of its simplicity.

For a single slender body, Sclavounos (1985*a*) has derived the relation

$$X_j^{\mathrm{S}} = \frac{\mathrm{i}\rho g}{2\omega}\int_L \mathrm{d}x\, q_j(x)\,\mathrm{e}^{-\mathrm{i}Kx\cos\beta} \quad (j = 3, 5), \tag{5.7}$$

where q_j is the outer source strength obtained by solving the unified-theory integral equation. The above equation is derived by applying the Haskind relations (Haskind 1957) to the far-field representation of the radiation velocity potential expressed in terms of a source distribution.

For twin slender bodies, the generalization of (5.7) takes the form

$$X_j^{\mathrm{T}} = \frac{\mathrm{i}\rho g}{\omega}\int_L \mathrm{d}x\, \mathrm{e}^{-\mathrm{i}Kx\cos\beta}\{[q_j(x)+q_{\mathrm{I},\,j}(x)]\cos(\tfrac{1}{2}KD\sin\beta) + \sin\beta\, d_{\mathrm{I},\,j}(x)\sin(\tfrac{1}{2}KD\sin\beta)\} \quad (j = 3, 5), \tag{5.8}$$

where q_j is the single-hull radiation source strength, and $q_{\mathrm{I},\,j}$, $d_{\mathrm{I},\,j}$ are the interaction source strength and dipole moment which come from solving the integral equations

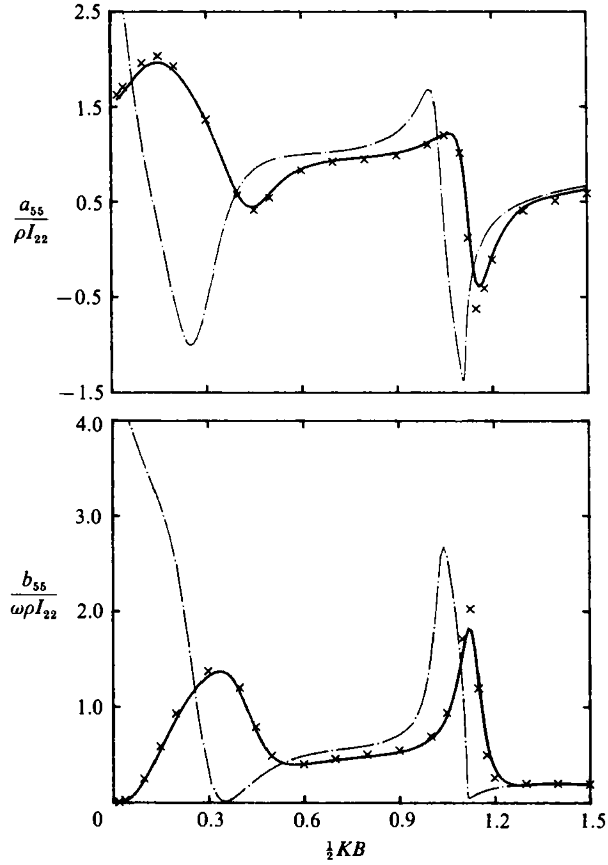

FIGURE 6. Pitch added moment of inertia and damping coefficients of twin spheroids with separation-to-length ratio $\frac{1}{2}$.

(4.14)–(4.15). The presence of the last term in the integrand of (5.8) is somewhat surprising because one might expect intuitively that the antisymmetric disturbance caused by a dipole distribution would not contribute to the symmetric-mode exciting forces. However, the dipole distributions on the two bodies oppose each other, so they create a symmetric disturbance which resembles a quadrupole in the far field.

6. Numerical results

To validate the far-field approximation, we have computed the heave and pitch hydrodynamic coefficients and exciting forces for twin spheroids whose major axes coincide with the free surface. The slenderness ratio of each spheroid is one-eighth and results are presented for separation distances of one-quarter and one-half the length of the spheroids. The far-field approximation is compared with strip theory and a three-dimensional boundary-integral formulation.

A brief description of the numerical scheme is given here. We use an algorithm developed by Nestegard & Sclavounos (1984) to compute the two-dimensional sway and heave coefficients of a circular cylinder over a range of frequencies. The algorithm also computes the two-dimensional source strength and dipole moment associated with heave and sway respectively. The hydrodynamic properties of any transverse

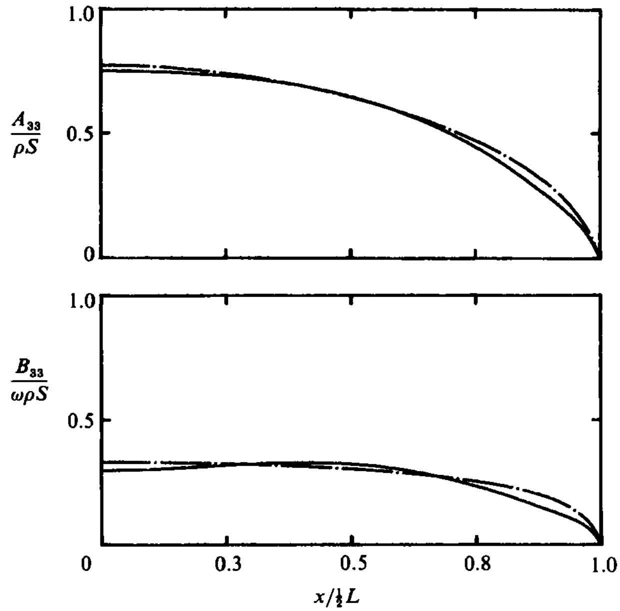

FIGURE 7. Section heave added-mass and damping for twin spheroids with separation-to-length ratio $\frac{1}{2}$ versus longitudinal position. Dimensionless wavenumber $\frac{1}{2}KB = 0.7$.

section of a single spheroid can be obtained by interpolating the data for a two-dimensional circular cylinder at the appropriate dimensionless frequency. The two-dimensional properties are needed at a finite number of transverse sections to solve the unified-theory integral equation for a single spheroid. The unified-theory solution in turn serves as the primary input to the far-field approximation.

The unified-theory integral equation and the far-field approximation (4.15) are solved by the collocation method. The longitudinal axis is divided into equal-length segments and the unknown source strength is approximated by quadratic B-splines with unknown coefficients. Each spline function spans three segments so that the continuity of the source strength and its first derivative are preserved. In most cases, 20 segments provide sufficient accuracy. A system of linear algebraic equations results from substituting the spline representation into the continuous form of either integral equation. This involves integrating the product of each spline function and the kernel of the integral equation. The singular terms in the unified-theory kernel are integrated analytically and the remaining regular terms are integrated numerically by Simpson's rule.

We first use the limiting case (4.17*a*, *b*) of the far-field approximation to compute the heave added-mass and damping coefficients of two semi-submerged circular cylinders and compare these results with a two-dimensional numerical solution developed by Sclavounos (1985*b*). The numerical solution is based on a boundary-integral formulation of the exact linear problem, includes all hydrodynamic interactions between the two cylinders, and has been verified through comparison with Wang & Wahab (1970) and Ohkusu (1970). The coefficients are plotted in figure 2, in dimensionless form, against the dimensionless wavenumber $\frac{1}{2}KB$ for distances between the cylinder centres of two and four times their diameter. Also shown for comparison are the corresponding coefficients of a single circular section. Two versions of the two-dimensional, far-field approximation have been tried. In the first,

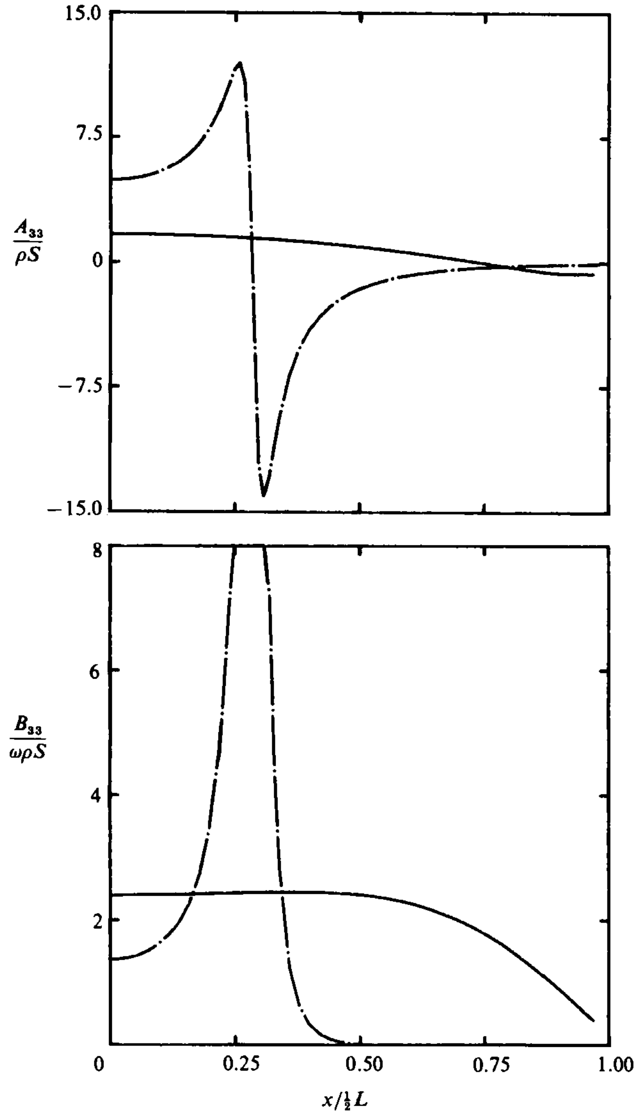

FIGURE 8. As figure 7 except $\frac{1}{2}KB = 1.1$.

the wave-source potential in (4.16) is replaced by an asymptotic approximation which is valid for large $K\,|y|$, while in the second its exact value is retained. The results from both methods are in very good agreement with the exact numerical solution for moderate-to-high frequencies, except near the lowest resonant frequency, and for a moderate separation distance. However the second method, which includes non-wavelike terms in the wave-source potential, performs substantially better than the first at low frequencies. This suggests that a similar improvement can be expected if the exact form of the three-dimensional wave-source potential is kept in (4.5) instead of its far-field asymptotic form.

Two distinct types of resonance can be observed in figure 2. The first type occurs near the wavenumbers $\frac{1}{2}KB \simeq 0.5$ and 0.25 for separation distances $d/B = 2.0$ and

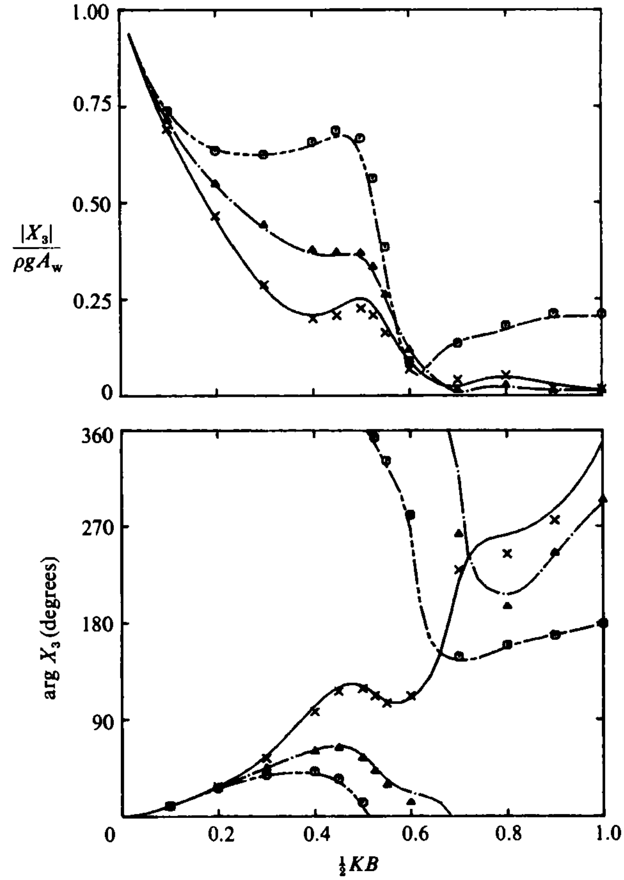

FIGURE 9. Modulus and phase of heave exciting force on twin spheroids with separation-to-length ratio $\frac{1}{4}$ according to far-field reciprocity relation at $\beta = 0$ (——), $\beta = 45$ (—·—), and $\beta = 90$ (-----). Corresponding tick marks denote three-dimensional numerical results. Modulus is non-dimensionalized by waterplane area A_w of twin spheroids.

4.0 respectively, and is attributable to a water-column-like behaviour of the enclosed portion of the free surface. This effect is noted by Marthinsen & Vinje (1985) and becomes more pronounced as the distance between the sections decreases. A second type of resonance occurs near the wavenumber $\frac{1}{2}KB = \frac{1}{3}\pi$ in the results for $D/B = 4$. The wavelength corresponding to this wavenumber equals the length $D-B$ of the enclosed free surface, so standing waves are present in the region between the two cylinders. There is of course an infinite number of higher wavenumbers where standing-wave resonance occurs. Although the behaviour of the coefficients may appear singular at the resonant wavenumbers, the peaks are actually finite because some wave energy leaks under the cylinders and radiates to the far field. Nevertheless, the elevation of the standing waves is quite large, making the validity of linear theory questionable in this regime.

We next compare results from the far-field approximation for twin spheroids with values from strip theory and a three-dimensional numerical solution. The numerical solution is described by Breit, Newman & Sclavounos (1985) and is based on a boundary-integral formulation of the exact linear problem. It has been validated

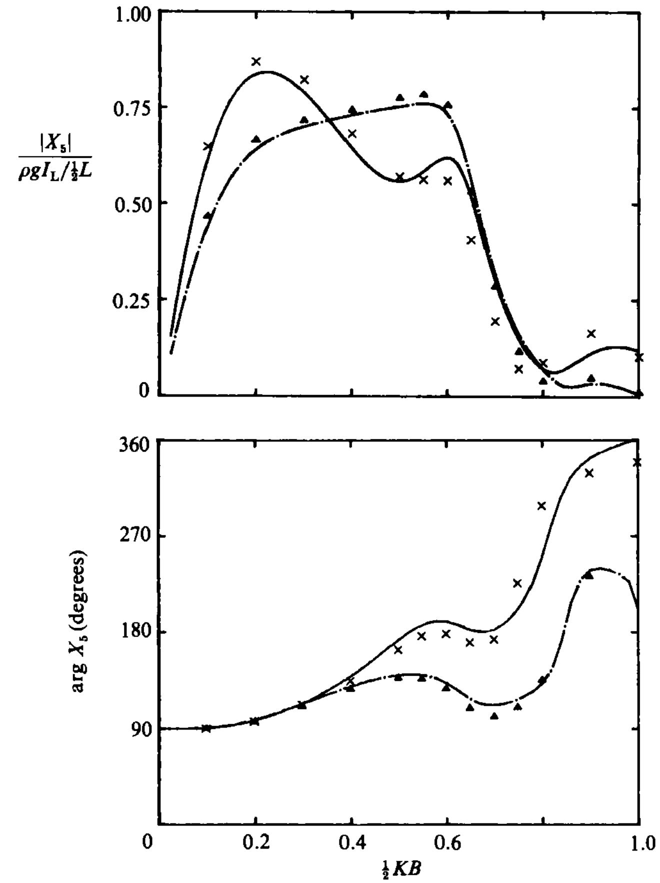

FIGURE 10. Modulus and phase of pitch exciting moment on twin spheroids with separation-to-length ratio $\frac{1}{4}$. Modulus is non-dimensionalized by longitudinal moment of inertia I_L of total water plane area.

through comparison with single-spheroid results from Kim (1965) and Yeung (1973), and we believe the results are accurate to three significant figures. We emphasize that the far-field approximation and this three-dimensional numerical solution are completely independent.

Heave and pitch added-mass and damping coefficients for a separation distance $D/L = \frac{1}{4}$ are shown in figures 3 and 4. For two reasons this case is actually a fairly severe test of the far-field approximation. First, the bodies are close together; the distance between their surfaces at the mid-section is only one diameter. Secondly, a spheroid has blunt ends which violate our initial assumption that both the source strength and its longitudinal derivative are smooth at the ends. Nevertheless, the far-field approximation is in very good agreement with the three-dimensional numerical results, even at fairly low wavenumbers. Since the far-field approximation performs so well for this case, we expect it to perform equally well, if not better, when the bodies are further apart or have more slender shapes at the ends.

The large discrepancy in the strip-theory results at low wavenumbers was anticipated since the same qualitative differences between strip theory and unified theory are present even for a single slender body. Strip theory is strictly valid only for short waves.

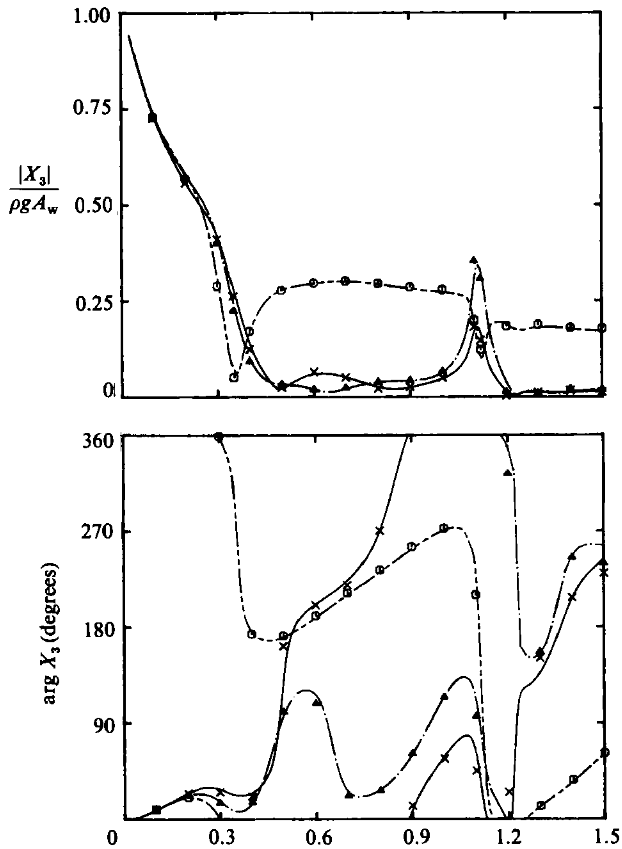

FIGURE 11. Heave exciting force on twin spheroids with separation-to-length ratio $\frac{1}{2}$.

Results for the case $D/L = \frac{1}{2}$ are presented in figures 5 and 6. Once again, the agreement between the far-field approximation and the numerical model is very good. The irregularity in the curve at $\frac{1}{2}KB = \frac{1}{3}\pi$ is evidence of resonance between the hulls. This wavenumber range corresponds to wavelengths which are nearly equal to the minimum distance between the hulls. It also coincides with the wavenumber where resonance occurs in the comparable two-dimensional interaction problem. Although the strip-theory results appear to behave quite well in the resonant regime, strip theory actually breaks down, as has been noted by Lee (1976). In figures 7 and 8, we show the longitudinal distribution of added-mass and damping according to strip theory and the far-field approximation at two wavenumbers. Away from resonance at $\frac{1}{2}KB = 0.7$ (figure 7), there are only minor differences between the two methods. However at $\frac{1}{2}KB = 1.1$ (figure 8), there is an irregularity in the strip-theory results in the vicinity of $x/L = 0.28$ (and an identical one at the reflection of this point about $x/L = 0$). This means that standing-wave resonance is occurring in the two-dimensional problem at that longitudinal location because strip theory assumes that wave interaction between the bodies is restricted to transverse planes. The rapid longitudinal variation in the flow is physically unrealistic and violates the underlying assumption of slender-body theories that the flow varies slowly in the longitudinal direction. The smooth distributions predicted by the far-field approximation indicate that it accounts for longitudinal interaction in a physically more reasonable manner.

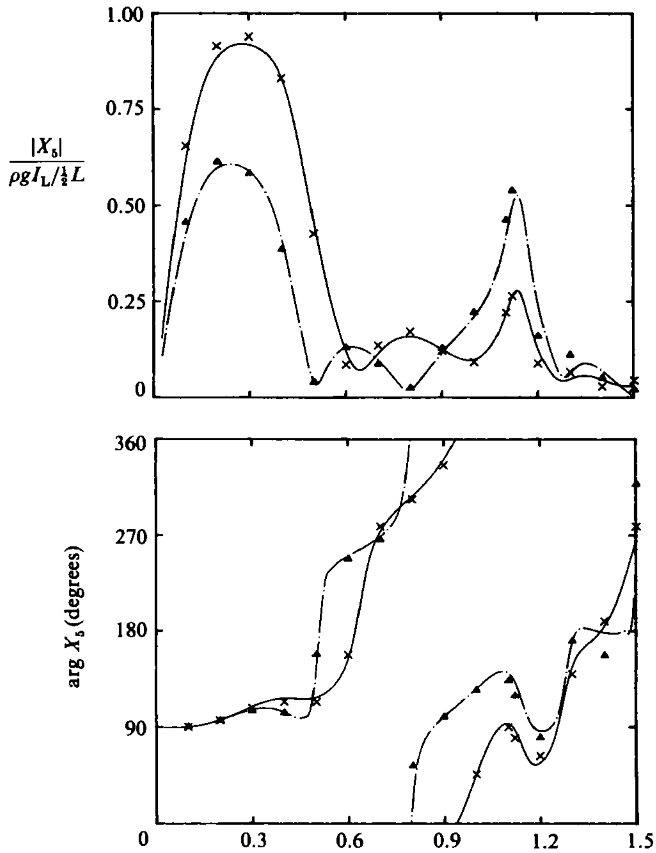

FIGURE 12. Pitch exciting moment on twin spheroids with separation-to-length ratio $\frac{1}{2}$.

Heave and pitch exciting forces for three wave directions have been calculated from the far-field reciprocity relation (5.8). The modulus and phase of the heave exciting force and pitch exciting moment are plotted in figures 9–12 together with results from the three-dimensional numerical method. The latter results are based on a direct solution of the diffraction problem rather than a reciprocity relation, and are believed to be accurate to at least two decimal places. Once again, the accuracy of the far-field approximation is remarkable.

7. Conclusions and extensions

The far-field approximation has been found to be very robust for all tested frequencies and for moderate separation distances. This is surprising because the initial assumption that the bodies are in the far field of each other appears to be invalid in this case. At higher frequencies where resonance occurs, the far-field approximation appears to be physically realistic while strip theory does not. The reciprocity relations for the exciting force appear to be equally reliable.

This approach can be easily adapted to problems involving interaction between two or more independent slender bodies. We have derived a system of coupled integral equations for the case of two independent slender bodies. A practical problem that can be treated in this manner is that of predicting the relative motions of two adjacent, independent ships. When multiple independent bodies are present, it is convenient to decompose the radiation problem into separate problems where each body undergoes a forced oscillation while all other bodies are fixed at their mean positions. Extensions to wave interactions between slender bodies in acoustics or electromagnetics can follow lines similar to the present method.

The authors wish to acknowledge the financial support provided by the National Science Foundation (Contract MEA 8210649) and the Office of Naval Research (Special Focus Program on Ship Hydrodynamics, Contract N0014-8222-K-0198). The first author has been supported by an Office of Naval Research Graduate Fellowship.

REFERENCES

BREIT, S. R., NEWMAN, J. N. & SCLAVOUNOS, P. D. 1985 A new generation of panel programs for radiation–diffraction problems. In *Proc. 4th Intl Conf. Behavior of Off-Shore Structures* (*BOSS '85*), *Delft* (ed. J. A. Battjes). Elsevier.

GREENHOW, M. J. L. 1980 The hydrodynamic interactions of spherical wave-power devices in surface waves. In *Power from Sea Waves* (ed. B. M. Count), pp. 287–343. Academic.

HASKIND, M. D. 1957 The exciting forces and wetting of ships in waves (in Russian). *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk* 7, 65–79. English translation available as David Taylor Model Basin Translation No. 307.

KAGEMOTO, H. & YUE, D. K. P. 1985 Wave forces on multiple leg platforms. In *Proc. 4th Intl Conf. Behavior of Off-Shore Structures* (*BOSS '85*), *Delft* (ed. J. A. Battjes). Elsevier.

KIM, W. D. 1965 On the harmonic oscillations of a rigid body on a free surface. *J. Fluid Mech.* **21**, 427–451.

KORVIN-KROUKOVSKY, B. V. 1955 Investigation of ship motions in regular waves. *Trans. Soc. Naval Archs and Mar. Engrs Trans.* **85**, 590–632.

LEE, C. M. 1976 Theoretical prediction of motion of small-waterplane-area, twin-hull (SWATH) ships in waves. *David W. Taylor Naval Ship Research and Development Center Rep.* 76-0046.

MARTHINSEN, T. & VINJE, T. 1985 Nonlinear hydrodynamic interaction in offshore loading systems. In *Proc. 4th Intl Conf. on Behavior of Off-Shore Structures* (*BOSS '85*), *Delft* (ed. J. A. Battjes). Elsevier.

MARTIN, P. A. 1984 Multiple scattering of surface water waves and the null-field method. In *Proc. 15th Symp. on Naval Hydrodynamics, Hamburg.*

MILES, J. W. 1983 Surface-wave diffraction by a periodic row of submerged ducts. *J. Fluid Mech.* **128**, 155–180.

NESTEGARD, A. & SCLAVOUNOS, P. D. 1984 A numerical solution of two-dimensional deep water wave-body problems. *J. Ship Res.* **24**, 8–23.

NEWMAN, J. N. 1978 The theory of ship motions. *Adv. Appl. Mech.* **18**, 221–283.

OGILVIE, T. F. & TUCK, E. O. 1969 A rational strip theory of ship motions. Part I. *Dept of Naval Architecture and Marine Engineering, University of Michigan, Ann Arbor Rep. No.* 013.

OHKUSU, M. 1970 On the heaving motion of two circular cylinders on the surface of a fluid. *Reports of Research Institute for Applied Mechanics, Kyushu Univ.*, Vol. XVII No. 58, pp. 167–185.

OHKUSU, M. 1974 Hydrodynamic forces on multiple cylinders in waves. In *Proc. Intl Symp. on the Dynamics of Marine Vehicles and Structures in Waves*, pp. 107–112. Institute of Mechanical Engineers.

SCLAVOUNOS, P. D. 1984 The diffraction of free-surface waves by a slender ship. *J. Ship Res.* **28**, 29–47.

SCLAVOUNOS, P. D. 1985*a* Forward speed vertical wave exciting forces on ships in waves. *J. Ship Res.* **29**, pp. 105–111.

SCLAVOUNOS, P. D. 1985*b* User manual of NIIRID: a general purpose program for wave–body interactions in two dimensions. *Dept of Ocean Engineering, Massachusetts Institute of Technology, Cambridge, Mass.*

SIMON, M. J. 1982 Multiple scattering in arrays of axisymmetric wave-energy devices. Part 1. A matrix method using a plane-wave approximation. *J. Fluid Mech.* **120**, 1–25.

SPRING, B. H. & MONKMEYER, P. L. 1974 Interaction of plane waves with vertical cylinders. In *Proc. 14th Intl Conf. on Coastal Engineering*, chapter 107, pp. 1828–1845.

SROKOSZ, M. A. & EVANS, D. V. 1979 A theory for wave-power absorption by two independently oscillating bodies. *J. Fluid Mech.* **90**, 337–362.

TWERSKY, V. 1952 Multiple scattering of radiation by an arbitrary configuration of parallel cylinders. *J. Acoust. Soc. Am.* **24**, 42–46.

WANG, S. & WAHAB, R. 1970 Heaving oscillations of twin cylinders in a free surface. *J. Ship Res.* **15**, 33–48.

WEHAUSEN, J. V. & LAITONE, E. V. 1960 Surface waves. In *Handbuch der Physic* (ed. S. Flugge), vol. 9, pp. 446–778.

YEUNG, R. W. 1973 A singularity-distribution method for free-surface flow problems with an oscillating body. *Rep. No.* NA 73–6, Col. of Engng, University of California, Berkeley.